Interaction corrections to tunneling conductance in ballistic superconductors

Alex Levchenko
Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
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It is known that in the two-dimensional disordered superconductors electron-electron interactions in the Cooper channel lead to the negative logarithmical in temperature correction to the tunneling conductance, $\delta g_{\mathrm{DOS}}\propto-\ln(\frac{T_c}{T-T_c})$, above the critical temperature T_c. Physically this result appears due to the density-of-states suppression by superconductive fluctuations near the Fermi level. It is interesting that the other correction, which accounts for the Maki-Thompson-type interaction of fluctuations, is positive and exhibits strong power law, $\delta g_{\mathrm{MT}}\propto(\frac{T_c}{T-T_c})^3$, which dominates the logarithmic term in the immediate vicinity of the critical temperature. An interplay between these two contributions determines the zero-bias anomaly in fluctuating superconductors. This Brief Report is devoted to the fate of such interaction corrections in the ballistic superconductors. It turns out that ballistic dynamic fluctuations perturb the single-particle density of states near the Fermi level at the energy scale $\epsilon\sim\sqrt{T_c(T-T_c)}$, which is different from $\epsilon\sim T-T_c$, relevant in the diffusive case. As the consequence, fluctuation region becomes much broader. In this regime we confirm that correction to the tunneling conductance remains negative and logarithmic not too close to the critical temperature while in the immediate vicinity of the transition we find different power law for the Maki-Thompson contribution, $\delta g_{\mathrm{MT}}\propto(\frac{T_c}{T-T_c})^{3/2}$. We suggest that peculiar nonmonotonous temperature dependence of the tunneling conductance may be probed via magnetotunnel experiments.

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As it is well known,[1] the leading-order fluctuation corrections to the conductivity due to electron-electron interaction in the Cooper channel in the vicinity of the superconducting transition are given by the Aslamazov-Larkin[2] (AL), Maki-Thompson[3,4] (MT), and density-of-states[5] (DOS) contributions. The first one has a simple physical meaning of the direct charge transfer mediated by fluctuation preformed Cooper pairs. The other two contributions have a purely quantum origin. The Maki-Thompson process can be understood as the coherent Andreev reflection of electrons on the local fluctuations of the order parameter while density of states effects originate from the depletion of energy states near the Fermi level by superconductive fluctuations. The relative importance of these three contributions depends on whether a superconductor is diffusive ($T\tau_{\mathrm{el}}\ll1$), ballistic ($T\tau_{\mathrm{el}}\gg1$), or granular ($\delta\ll\Gamma\ll\max\{E_{Th},T\}$). Here τ_{el} is the elastic-scattering time on impurities, δ is the mean level spacing in the grain, Γ is the escape rate, $E_{Th}=D/\ell^2$ is Thouless energy for a grain with the typical size ℓ, and D is the diffusion coefficient. The Aslamazov-Larkin correction is essential in both pure and impure superconductors. The Maki-Thompson is important only in the disordered systems since there is strong cancellation between MT and density-of-states effects in the ballistic regime.[6] Usually unimportant DOS contributions become crucial in the systems containing tunneling junctions[7,8] or in granular superconductors.[9,10] Tunnel barriers or granularity require multiple electron scattering for AL and MT contributions to be important. As the result, the magnitude of these effects is suppressed either by an extra powers of tunneling matrix element $\sim|t_{pk}|^2$ (in the case of tunnel barriers[7]) or by the small ratio $g_\Gamma/g_\delta\ll1$ between intergrain $g_\Gamma\sim\Gamma/\delta$ and intragrain $g_\delta\sim E_{Th}/\delta$ conductances (in the case of granular superconductors[9,10]).

In the study[7] of tunneling anomaly between diffusive thin-film superconductors separated by an insulating layer it was shown that there is one specific MT-type process that contributes significantly to the renormalization of the tunneling conductance. This process appears to the first order in tunneling probability $|t_{pk}|^2$, like DOS contribution, however, to the second order in interaction, unlike DOS, thus containing one extra power of small Ginzburg number, $Gi\lesssim1$. The reason why these two contributions have to be accounted simultaneously is twofold. Unlike DOS correction δg_{DOS}, which leads to the suppression of the tunneling conductance above the critical temperature T_c, the MT-type contribution δg_{MT} leads to its enhancement. Second, although being smaller by the inverse power of dimensionless conductance $g=\nu D=k_F\ell_{\mathrm{el}}/2\pi\gg1$, where $\nu=m/\pi$ is the single-particle density of states in two dimensions and $\ell_{\mathrm{el}}=v_F\tau_{\mathrm{el}}$, this specific MT contribution has much stronger power-law temperature dependence, $\delta g_{\mathrm{MT}}\propto Gi^2(\frac{T_c}{T-T_c})^3$, as opposed to the weak logarithmic in temperature correction coming from the density of states, $\delta g_{\mathrm{DOS}}\propto-Gi\ln(\frac{T_c}{T-T_c})$. One should recall here that the parameter that controls perturbative expansion over superconductive fluctuations is set by the Ginzburg number, which is just inversely proportional to the dimensionless conductance, $Gi\propto1/g$. So that, this is really a competition between these two contributions that defines the nature of zero-bias anomaly in fluctuating superconductors. As a result, due to an opposite signs of δg_{DOS} and δg_{MT} terms the full conductance correction $\delta g_{\mathrm{DOS}}+\delta g_{\mathrm{MT}}$ has nonmonotonous temperature dependence that even may change sign. Similar observations emerge in the context of granular superconductors.[10]

The reason for such strong temperature dependence of δg_{MT} was attributed in Refs. 7 and 10 to the importance of vertex renormalization by coherent impurity scattering (Cooper ladders). If so this would imply then that anomalous Maki-Thompson contribution is absent in ballistic tunnel junctions. Such conclusion is also appealing in the view of strong cancellation of MT effects in ballistic regime of su-

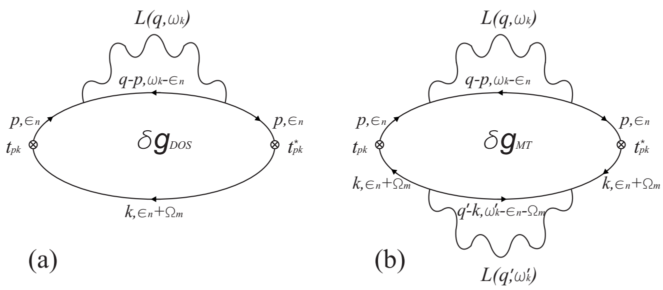

FIG. 1. DOS and MT tunneling conductance correction diagrams.

perconducting thin films.[6] However, as we show in this work, in contrast to the expectation MT interaction correction to conductance in ballistic tunnel junctions remains important. It also exhibits strong temperature dependence, similar to that in the diffusive regime, but with the fractional powers depending on dimensionality.

In what follows, we carry out a microscopic calculation of interaction corrections to the tunneling conductance in ballistic superconductors $T\tau_{\mathrm{el}}\gg 1$ with the help of standard temperature diagrammatic technique.[11] Within this formalism the conductance

$$g_T=-e\frac{\partial}{\partial V}\mathrm{Im}[\Pi^R(\Omega)]_{\Omega=eV} \tag{1}$$

is determined by the retarded component of the polarization operator $\Pi^R(\Omega)$. Here e is the electron charge and V is the voltage applied across the junction. In the case of noninteracting electrons Matsubara version of the polarization operator is given by the simple loop diagram, which reads analytically as $\Pi(\Omega_m)=T\Sigma_{\epsilon_n}\Sigma_{pk}|t_{pk}|^2G(p,\epsilon_n+\Omega_m)G(k,\epsilon_n)$, where t_{pk} stands for the tunneling matrix element, $\epsilon_n=2\pi T(n+1/2)$ and $\Omega_m=2\pi Tm$ are fermionic and bosonic Matsubara frequencies, respectively, and

$$G(p,\epsilon_n)=\frac{1}{i\epsilon_n-\xi_p+\dfrac{i\,\mathrm{sgn}\,\epsilon_n}{2\tau_{\mathrm{el}}}},\qquad \xi_p=\frac{p^2-p_F^2}{2m} \tag{2}$$

defines the single-particle Green's function. Under the assumption of momentum-independent tunneling amplitudes a simple calculation then gives for the bare value of the conductance $g_T=\frac{\pi}{2}e^2\nu^2|t_{pk}|^2$. The first-order interaction correction is given by the diagram shown in Fig. 1(a)

$$\delta\Pi_{\mathrm{DOS}}(\Omega_m)=2T^2\sum_{\epsilon_n\omega_k}\sum_{pkq}|t_{pk}|^2G^2(p,\epsilon_n)G(k,\epsilon_n+\Omega_m)\times G(q-p,\omega_k-\epsilon_n)L(q,\omega_k), \tag{3}$$

which amounts an insertion of a single interaction line into one of the Green's function and coefficient of two accounts for two such possibilities. This is the density of states effect since upper part of the diagram is just a self-energy for the $G(p,\epsilon_k)$. The interaction propagator is defined as

$$L(q,\omega_k)=-\frac{8T}{\pi\nu}\frac{1}{Bq^2+\tau_{\mathrm{GL}}^{-1}+|\omega_k|}, \tag{4}$$

where $B=\frac{7\zeta(3)v_F^2}{2d\pi^3T}$ and $\tau_{\mathrm{GL}}=\frac{\pi}{8(T-T_c)}$ with $d=1,2,3$ being effective dimensionality of a superconductor ($1d$ wire, $2d$ film, or $3d$ bulk). This approximate form of the interaction is obtained from the general expression[1] $L^{-1}(q,\omega)=-\nu[\ln\frac{T}{T_c}+\psi(\frac{1}{2}+\frac{|\omega_k|}{4\pi T})+\xi^2(T\tau_{\mathrm{el}})q^2-\psi(\frac{1}{2})]$, where $\xi(T\tau_{\mathrm{el}})=\frac{v_F^2\tau_{\mathrm{el}}^2}{d}[\psi(\frac{1}{2})+\frac{1}{4\pi T\tau_{\mathrm{el}}}\psi'(\frac{1}{2})-\psi(\frac{1}{2}+\frac{1}{4\pi T\tau_{\mathrm{el}}})]$, under the assumption that characteristic energies of fluctuations are small as compared to the temperature, $\omega\sim T-T_c\ll T$, and momenta are small as compared to the inverse thermal length $\ell_T=v_F/T$, $q\sim\sqrt{1/B\tau_{\mathrm{GL}}}\ll\ell_T^{-1}$, which allows to expand digamma functions ψ at small argument.

Since matrix elements t_{pk} depend weakly on the momenta near the Fermi-surface one can substitute summation over p and k in Eq. (3) by the corresponding integration over the energies $\Sigma_{pk}(\cdots)\Rightarrow\frac{g_T}{4\pi e^2}\int_{-\infty}^{+\infty}d\xi_pd\xi_k(\cdots)$. Once these integrations are performed assuming ballistic limit $\max\{\epsilon_n,\omega_k\}\gg\tau_{\mathrm{el}}^{-1}$ and approximating $\xi_{q-p}\approx\xi_p-v_F\cdot q$

$$\sum_{pk}|t_{pk}|^2G^2(p,\epsilon_n)G(k,\epsilon_n+\Omega_m)G(q-p,\omega_k-\epsilon_n)\approx-\frac{\pi g_T}{4e^2}\frac{\mathrm{sgn}(\epsilon_n)\mathrm{sgn}(\epsilon_n+\Omega_m)\theta[\epsilon_n(\epsilon_n-\omega_k)]}{(v_F\cdot q+i\omega_k-2i\epsilon_n)^2}, \tag{5}$$

where $\theta(x)$ is the step function, one can complete summation over the bosonic frequency ω_k in Eq. (3) by converting it into the contour integral and make an analytical continuation $i\epsilon_n\to\epsilon+i0$. By combining the result for $\delta\Pi_{\mathrm{DOS}}$ with Eq. (1) one obtains density-of-states-type correction to the zero-bias conductance

$$\frac{\delta g_{\mathrm{DOS}}}{g_T}=\mathrm{Im}\sum_q\int_{-\infty}^{+\infty}\frac{d\epsilon}{2T\cosh^2\dfrac{\epsilon}{2T}}\times\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{L^K(q,\omega)+L^R(q,\omega)\tanh\dfrac{\epsilon-\omega}{2T}}{(\omega+v_F\cdot q-2\epsilon_+)^2}, \tag{6}$$

where we introduced Keldysh component of the interaction propagator $L^K(q,\omega)=[L^R(q,\omega)-L^A(q,\omega)]\coth\frac{\omega}{2T}$ while the retarded/advances components $L^{R(A)}(q,\omega)$ are obtained from Eq. (4) by the replacement $|\omega_k|\to\mp i\omega$. The most singular in $T-T_c$ contribution to δg_{DOS} comes from the branch-cut of $L^K(q,\omega)$ where $L^R(q,\omega)\tanh\frac{\epsilon-\omega}{2T}$ term can be ignored. By taking $L^K(q,\omega)\approx-(32iT_c^2/\pi\nu)[(Bq^2+\tau_{\mathrm{GL}})^2+\omega^2]^{-1}$ one can complete energy integration in Eq. (6) and find

$$\frac{\delta g_{\mathrm{DOS}}}{g_T}=\frac{4}{\pi^3\nu}\mathrm{Re}\sum_q\frac{\psi''\left(\dfrac{1}{2}+\dfrac{Bq^2+iv_F\cdot q+\tau_{\mathrm{GL}}^{-1}}{4\pi T}\right)}{Bq^2+\tau_{\mathrm{GL}}^{-1}}, \tag{7}$$

where $\psi''(x)$ is the second derivative of the digamma function. The remaining q sum is dominated by the small momentum transfer where argument of the digamma function can be taken as the constant $\psi''(1/2)=-14\zeta(3)$ since

$\max\{Bq^2, v_F q, \tau_{\mathrm{GL}}\} \ll T$. One obtains then as the result[12]

$$\frac{\delta g_{\mathrm{DOS}}}{g_T} = -a_d \begin{cases} C_1\sqrt{T_c\tau_{\mathrm{GL}}} & 1d, \\ C_2\ln(T_c\tau_{\mathrm{GL}}) & 2d, \end{cases} \tag{8}$$

where dimensionless prefactors are $C_1 = \frac{1}{\nu S\sqrt{BT_c}} \propto \frac{1}{p_F^2 S}$ and $C_2 = \frac{1}{\nu Bh} \propto \frac{1}{p_F h}\frac{T_c}{\epsilon_F}$, with S being the cross-section area of the wire and h being the thickness of the film. The numerical coefficients are $a_1 \approx 2.17$ and $a_2 \approx 0.17$. For the bulk $3d$ junctions $\delta g_{\mathrm{DOS}}/g_T \propto -C_3 = \frac{T_c}{\nu v_F B}$ is small and temperature independent. Notice also that for $2d$ case C_2 is linearly proportional to the Ginzburg number. One sees from Eq. (8) that strong suppression in the density of states near the Fermi level translates only into moderate renormalization of conductance δg_{DOS}. This observation brings us to the necessity to study contributions to conductance coming form the interacting fluctuations shown diagrammatically in Fig. 1(b). The reason for this is similar to that in the diffusive regime. First of all, this contribution is of the same order in tunneling $\sim |t_{pk}|^2$ as the density-of-states one, Fig. 1(a). Second, although having an extra small prefactor, C_d, this contribution is positive, unlike δg_{DOS}, and has much stronger temperature dependence, which may dominate δg_{DOS} in the near vicinity of the critical temperature. The competition between these terms defines the nature of zero-bias anomaly in fluctuating regime of ballistic superconductors.

The diagram in Fig. 1(b) defines Maki-Thompson correction to the polarization operator, which reads explicitly as

$$\begin{aligned}\delta\Pi_{\mathrm{MT}}(\Omega_m) = T^3 \sum_{\epsilon_n\omega_k\omega_k'} \sum_{pkqq'} & G^2(p,\epsilon_n)G(q-p,\omega_k-\epsilon_n) \\ & \times G^2(k,\epsilon_n+\Omega_m)G(q'-k,\omega_k'-\epsilon_n) \\ & \times L(q,\omega_k)L(q',\omega_k').\end{aligned} \tag{9}$$

It is important to comment here that although this correction looks like second-order DOS, it in fact contains the mixture of advanced and retarded blocks of the Green's functions, which by its analytical structure is the same as in the Maki-Thompson diagram. This is precisely the reason why this term is strongly temperature dependent. DOS effects always involve Green's functions of the same causality and thus bring subleading temperature dependence. One calculates momentum integrals in Eq. (9) by the prescription defined in Eq. (5) and after the analytical continuation finds corresponding correction to the conductance

$$\begin{aligned}\frac{\delta g_{\mathrm{MT}}}{g_T} = & -\frac{1}{2\pi^2 T}\mathrm{Re}\sum_{qq'}\int_{-\infty}^{+\infty}\frac{d\epsilon}{\cosh^2\frac{\epsilon}{2T}}\int\int_{-\infty}^{+\infty} d\omega d\omega' \\ & \times \frac{\mathrm{Im}[L^R(q,\omega)]\mathrm{Im}[L^A(q',\omega')]\coth\left(\frac{\omega}{2T}\right)\coth\left(\frac{\omega'}{2T}\right)}{(v_F\cdot q+\omega-2\epsilon_+)^2(v_F\cdot q'-\omega'+2\epsilon_-)^2},\end{aligned} \tag{10}$$

where $\epsilon_\pm = \epsilon \pm i0$ stands as the reminder of analyticity. After the consecutive energy integrations this formula simplifies to

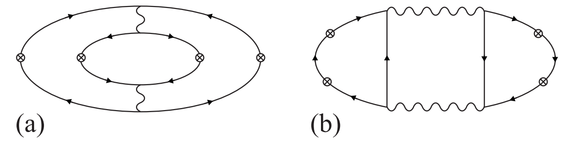

FIG. 2. Aslamazov-Larkin interaction corrections to the tunneling conductance which appear in the higher order in transparency $\sim |t_{pk}|^4$ then DOS and MT contributions shown in Fig. 1.

$$\begin{aligned}\frac{\delta g_{\mathrm{MT}}}{g_T} = & \frac{256T_c^3}{\pi\nu^2}\mathrm{Re}\sum_{qq'}\frac{1}{(Bq^2+\tau_{\mathrm{GL}}^{-1})(Bq'^2+\tau_{\mathrm{GL}}^{-1})} \\ & \times\frac{1}{(Bq^2+Bq'^2+iv_F\cdot q+iv_F\cdot q'+2\tau_{\mathrm{GL}}^{-1})^3}.\end{aligned} \tag{11}$$

As compared to the corresponding result in the diffusive case[7] the interesting feature here is appearance of the $v_F\cdot q$ factors, which limits the phase space for the momentum transfer and in a way changes power-law behavior of the singular term in the conductance. The remaining momentum integration can be completed in the closed form and gives

$$\frac{\delta g_{\mathrm{MT}}}{g_T} = b_d C_d^2 (T_c\tau_{\mathrm{GL}})^{(7-2d)/2}, \tag{12}$$

for $d=1,2,3$ with $b_1=0.06$, $b_2=1.4\times10^{-3}$, and $b_3=4\times10^{-3}$. As anticipated δg_{MT} is positive and has much stronger power-law temperature dependence then δg_{DOS} near T_c. We conclude here that anomalous temperature dependence of δg_{MT} known from the impurity vertex renormalization in the diffusive case[7] survives in the ballistic regime as well, however, with the fractional powers of $T-T_c$.

There are two more diagrams in the second order in interaction that contribute to the conductance renormalization. These are shown in Fig. 2 and define Aslamazov-Larkin corrections. However, unlike DOS and MT terms in Fig. 1 these contributions appear only to the second order in the tunneling transparency and thus contain and extra smallness $\sim |t_{pk}|^4$. An estimate for these diagrams for two-dimensional case gives $\delta g_{\mathrm{AL}}/g_T \propto |t_{pk}|^2 C_2^2 (T_c\tau_{\mathrm{GL}})^{1/2}$, which has smaller amplitude and weaker temperature dependence then δg_{MT} in Eq. (12).

There is simple physical picture which allows to understand these results at the qualitative level. The current in a tunnel junction is determined by the product of the density of states convoluted with the difference of Fermi function, which measure occupation of the given state, namely, $I(V) \sim \int d\epsilon [f_F(\epsilon+eV)-f_F(\epsilon)]\nu(\epsilon+eV)\nu(\epsilon)$. Within the linear response one can identify then from $I(V)$ the zero-bias DOS: $\delta g_{\mathrm{DOS}}/g_T \sim \int \frac{\delta\nu(\epsilon)}{\nu}\cosh^{-2}(\frac{\epsilon}{2T})\frac{d\epsilon}{2T}$ and MT: $\delta g_{\mathrm{MT}}/g_T \sim \int \frac{\delta\nu^2(\epsilon)}{\nu^2}\cosh^{-2}(\frac{\epsilon}{2T})\frac{d\epsilon}{2T}$ conductance corrections. Thus, estimation of the temperature dependence of δg_{DOS} and δg_{MT} requires knowledge of the detailed structure of the density of states above T_c. To this end, let us understand at which energy window $\delta\epsilon$ superconductive fluctuations deplete single-particle energy states near the Fermi level and what is the depth of this suppression. The energy scale can be estimated knowing the time τ_ξ needed for the superconductive fluctuation to spread over the distance of coherence length $\xi(T)$

$=\xi_0\sqrt{\frac{T_c}{T-T_c}}$. In the disordered case τ_ξ is determined by the diffusive motion of particles and gives for $\delta\epsilon$ via the uncertainty relation $\delta\epsilon\sim\tau_\xi^{-1}=D\xi^{-2}(T)=\tau_{\rm GL}^{-1}\propto T-T_c$. In the clean limit ballistic motion defines another scale[12] $\delta\epsilon\sim\tau_\xi=v_F\xi^{-1}(T)\propto\sqrt{T_c(T-T_c)}$. The depth of the depletion region in DOS, $\delta\nu(\epsilon)=-\frac{1}{\pi}\mathrm{Im}\,\Sigma^R(\epsilon)$, follows from the self-energy of the electron Green's function $\Sigma(\epsilon_n)=T\Sigma_{pk\omega_n}G^2(p,\epsilon_n)G(q-p,\omega_k-\epsilon_n)\Gamma^2(q,\epsilon_n,\omega_k-\epsilon_n)L(q,\omega_k)$, where impurity vertex $\Gamma(q,\epsilon_n,\epsilon_m)=\tau_{\rm el}^{-1}\theta(-\epsilon_n\epsilon_m)/(Dq^2+|\epsilon_n-\epsilon_m|)$ is present only in the diffusive limit. Having calculated $\Sigma^R(\epsilon)$ DOS renormalization reads $\delta\nu_{D(B)}(\epsilon)\propto(T_c\tau_{\rm GL})^{(6-d)/2}F_{D(B)}(2\epsilon\tau_{\rm GL})$, where $F_{D(B)}(2\epsilon\tau_{\rm GL})$ are energy depending scaling functions which are universal for the given dimensionality. For example, in the diffusive $2d$ case[5]

$$F_D(z)=\frac{1}{1+z^2}+\frac{(1-z^2)}{2(1+z^2)^2}\ln\left(\frac{1+z^2}{4}\right)-\frac{2z\arctan(z)}{(1+z^2)^2} \qquad (13)$$

while in the ballistic regime[12]

$$F_B(z)=\frac{1}{z^2+\varkappa}\left[1-\frac{z}{\sqrt{z^2+\varkappa}}\ln\left(\frac{z+\sqrt{z^2+\varkappa}}{\sqrt{\varkappa}}\right)\right], \qquad (14)$$

where $\varkappa=[\pi^3/7\zeta(3)]T_c\tau_{\rm GL}$. The two basic properties of the scaling functions are $F_D(\epsilon\to0)\to\text{const}$ while $F_B(\epsilon\to0)\to1/\varkappa$ which is actually valid for any dimensionality and also $\int_{-\infty}^{+\infty}F_{D(B)}(\epsilon)d\epsilon=0$. The latter is the manifestation of conservation law for the total number of states. Knowing these facts one readily estimates $\delta\nu_D(0)\propto(T_c\tau_{\rm GL})^{(6-d)/2}$ and $\delta\nu_B(0)\propto(T_c\tau_{\rm GL})^{(4-d)/2}$. The most singular contribution to the MT conductance renormalization comes from the energy region of maximally depleted $\delta\nu(\epsilon)$ where interaction of superconductive fluctuations is the strongest. The width of this region is roughly $\delta\epsilon$ and, thus, interaction correction may be estimated as $\delta g_{\rm MT}/g_T\propto\delta\nu^2(0)\delta\epsilon$. For the diffusive case this gives

$$\frac{\delta g_{\rm MT}}{g_T}\propto(T_c\tau_{\rm GL})^{2\times(6-d)/2}(T_c\tau_{\rm GL})^{-1}\propto\left(\frac{T_c}{T-T_c}\right)^{5-d}, \qquad (15)$$

which reproduces results of Ref. 7 while in the ballistic case

$$\frac{\delta g_{\rm MT}}{g_T}\propto(T_c\tau_{\rm GL})^{2\times(4-d)/2}(T_c\tau_{\rm GL})^{-1/2}\propto\left(\frac{T_c}{T-T_c}\right)^{(7-2d)/2}, \qquad (16)$$

which agrees with our explicit diagrammatic calculation [Eqs. (9)–(12)]. The reason why $\delta g_{\rm DOS}$ remains logarithmic in both cases is due to the conservation law $\int_{-\infty}^{+\infty}\delta\nu_{D(B)}(\epsilon)d\epsilon=0$. Indeed, when performing energy integration in $\delta g_{\rm DOS}/g_T\sim\int\frac{\delta\nu(\epsilon)}{\nu}\cosh^{-2}(\frac{\epsilon}{2T})\frac{d\epsilon}{2T}$ one necessarily accounts for the pole of the Fermi function which set the relevant energies to be of the order of T and not $T-T_c$. For $\epsilon\sim T$ both scaling functions $F_{D(B)}(\epsilon)$ coincide to the leading singular order in $T-T_c$.

The possible way to probe these temperature anomalies in the conductance above T_c may be via magnetotunneling. Let us recall that magnetic field H acts as an effective Cooper pair breaking factor that drives a superconductor away from the critical region. As the result, the relevant energies ω that determine conductance corrections in Eqs. (6) and (10) are set by the largest cutoff between inverse Ginzburg-Landau time $\tau_{\rm GL}^{-1}\sim T-T_c$ and cyclotron frequency $\omega_H\propto H$, namely, $\omega\sim\max\{\omega_H,\tau_{\rm GL}^{-1}\}$. So that by changing the field one effectively traces temperature dependence of δg. Although an explicit calculations of magnetoconductance in ballistic superconductors is quite involved task we rely here on the plausible suggestion that is based on the results known for the diffusive case.[13] One may expect logarithmic in magnetic field dependence for the DOS correction $\delta g_{\rm DOS}(H)/g_T\propto-\ln(T_c/\omega_H)$, when $\omega_H\gtrsim\tau_{\rm GL}^{-1}$ and a power law of H for $\delta g_{\rm MT}(H)$.

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